2.1 Sejam

$$x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$$

e

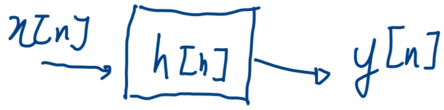
$$h[n] = 2\delta[n+1] + 2\delta[n-1].$$

Calcule e represente graficamente cada uma das convoluções a seguir

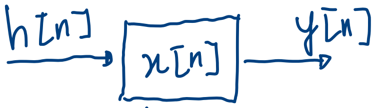
(a) $y_1[n] = x[n] * h[n]$

(b) $y_2[n] = x[n+2] * h[n]$

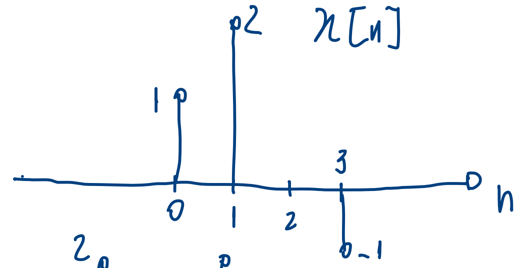
(c) $y_3[n] = x[n] * h[n+2]$

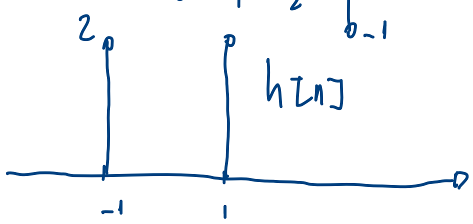


$$y[n] = \sum_{k=-\infty}^{+\infty} x[n]\, h[n-k]$$



$$y[n] = \sum_{k=-\infty}^{+\infty} h[n]\, x[n-k]$$

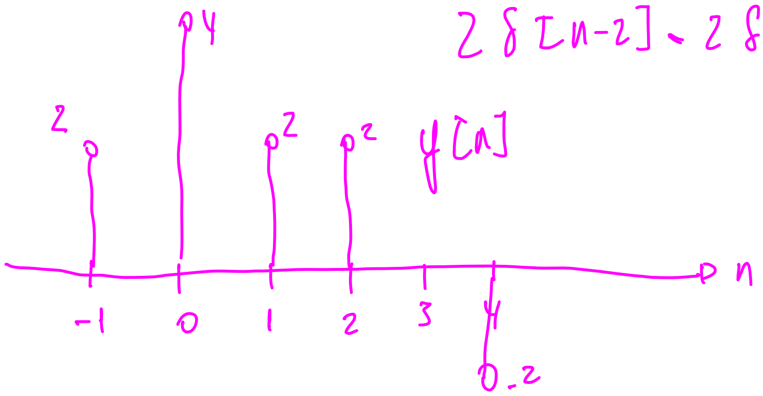




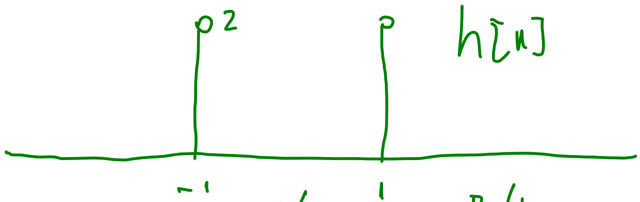
$$(a)\ y[n] = h[n] + 2h[n-1] - h[n-3]$$

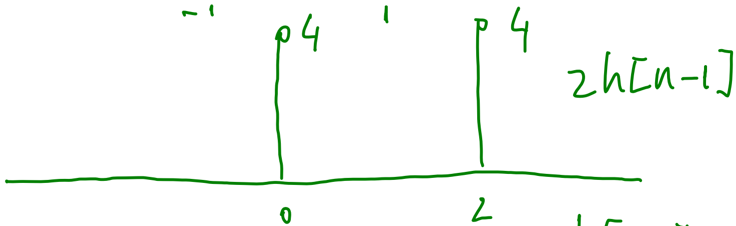
$$= 2\delta[n+1] + 2\delta[n-1] + 4\delta[n] + 4\delta[n-2] - 2\delta[n-2] - 2\delta[n-4]$$

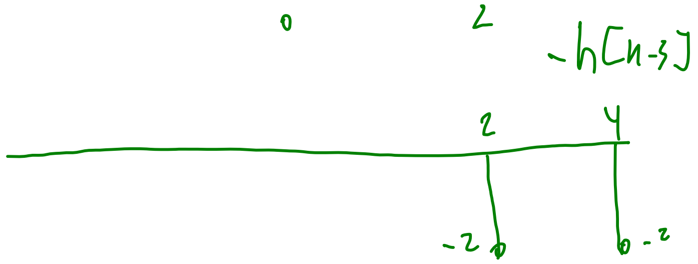
$$y[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$$

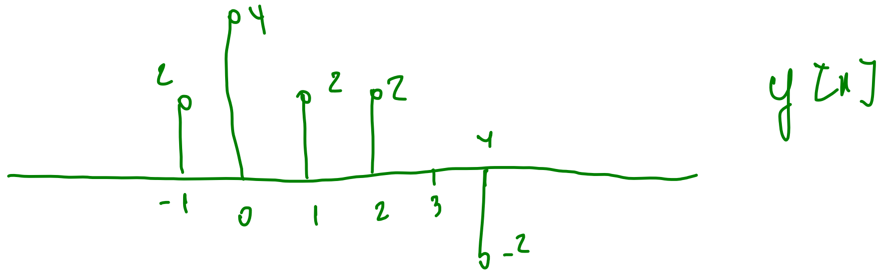


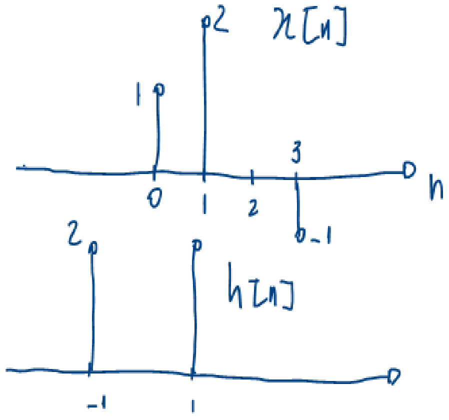
$$h[n] + 2h[n-1] - h[n-3] \qquad h[n] = 2\delta[n+1] + 2\delta[n-1]$$



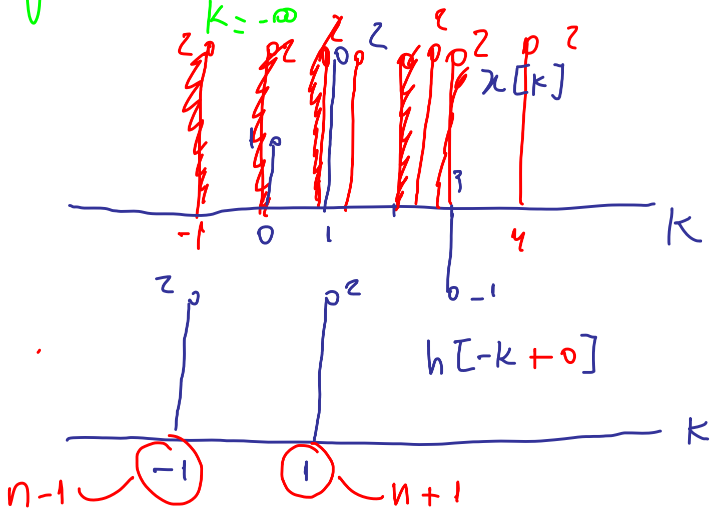








$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]$$



$n=-1 \quad y[-1]=2$

$n=0 \quad y[0]=4$

$n=1 \quad y[1]=2$

$n=2 \quad y[2]=2$

$n=3 \quad y[3]=0$

$n=4 \quad y[4]=-2$

$$y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[n-4]$$

(b) $y_2[n] = y_1[n+2]$

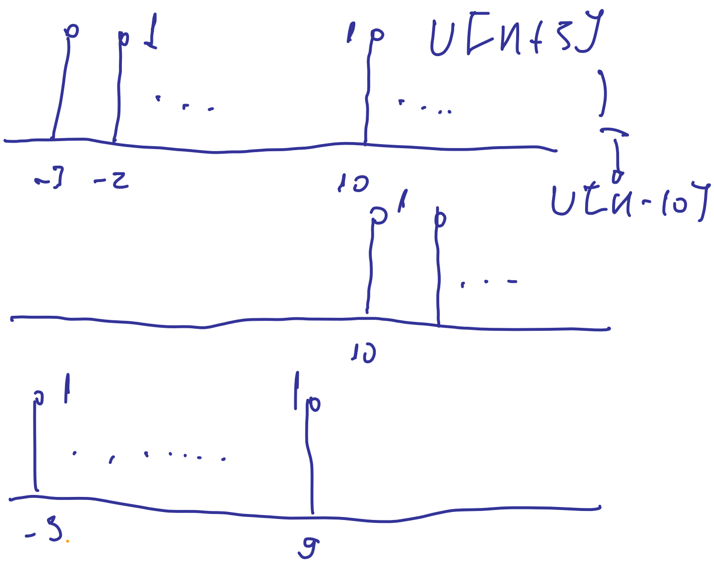
(c) $y_3[n] = y_2[n]$

2.2 Considere o sinal

$$h[n] = \left(\frac{1}{2}\right)^{n-1}\{u[n+3] - u[n-10]\}.$$

Expresse A e B em termos de n de modo que a seguinte equação seja válida:

$$h[n-k] = \begin{cases} \left(\frac{1}{2}\right)^{n-k-1}, & A \le k \le B \\ 0, & \text{caso contrário} \end{cases}.$$



$$h[n] = \begin{cases} \left(\frac{1}{2}\right)^{n-1}, & -3 \le n \le 9 \\ 0, & \text{c.c} \end{cases}$$

$$h[n-k] = \begin{cases} \left(\frac{1}{2}\right)^{n-k-1}, & -3 \le n-k \le 9 \\ 0, & \text{c.c} \end{cases}$$

$-3-n \le -k \le 9-n$

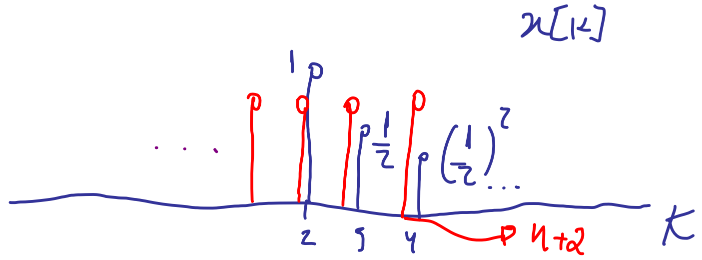
$3+n \ge k \ge n-9 \rightarrow \overset{A}{n-9} \le k \le \overset{B}{n+3}$

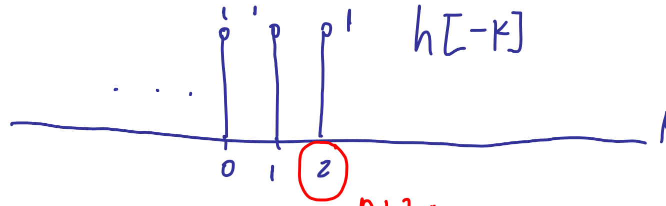
2.3 Considere uma entrada $x[n]$ e uma resposta ao impulso unitário $h[n]$ dadas por

$$x[n] = \left(\frac{1}{2}\right)^{n-2} u[n-2],$$

$$h[n] = u[n+2].$$

Determine e represente graficamente a saída $y[n] = x[n] * h[n]$.





$n = 0 \quad y[0] = 1$

$y[1] = 1 + \frac{1}{2}$

$y[2] = 1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2$

$y[n] = \left[\sum_{k=0}^{n} \left(\frac{1}{2}\right)^k\right] u[n]$

$$y[n] = \sum_{k=\cancel{-\infty}\,2}^{\cancel{+\infty}\,n+2} \underbrace{x[k]}_{\left(\frac{1}{2}\right)^{k-2}} \underbrace{h[n-k]}_{1}$$

$$y[n] = \sum_{k=2}^{n+2} \left(\frac{1}{2}\right)^{k-2}$$

$$\left[\sum_{k=0}^{n} \left(\frac{1}{2}\right)^k\right] u[n]$$

$$\sum_{0}^{n} = \sum_{0}^{\infty} - \sum_{n+1}^{\infty}$$

$$\frac{1}{1-\frac{1}{2}} - \frac{\left(\frac{1}{2}\right)^{n+1}}{1-\frac{1}{2}} = \left[2 - 2\left(\frac{1}{2}\right)^{n+1}\right] u[n] = y[n]$$

$$2\left(1 - \left(\frac{1}{2}\right)^{n+1}\right) u[n]$$