

Sinais e Sistemas

Sistemas Lineares Invariantes no Tempo

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Fundação Educacional Montes Claros



Integral de Convolução

Exemplo

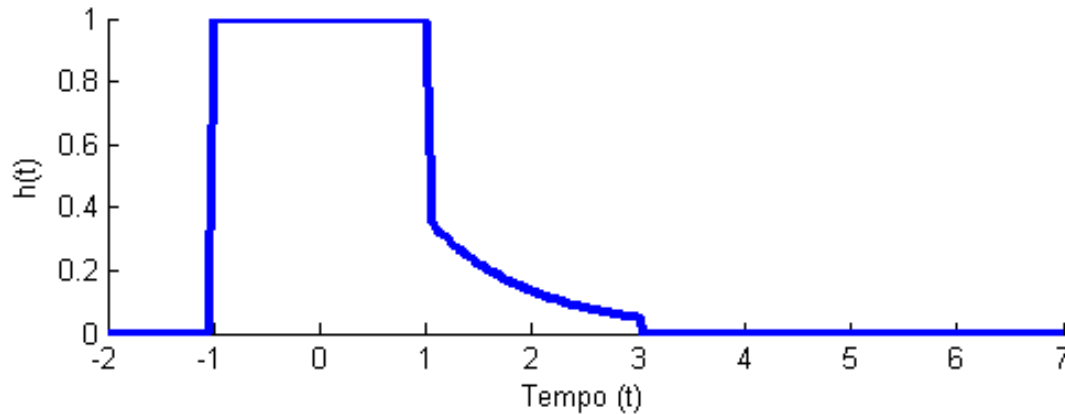
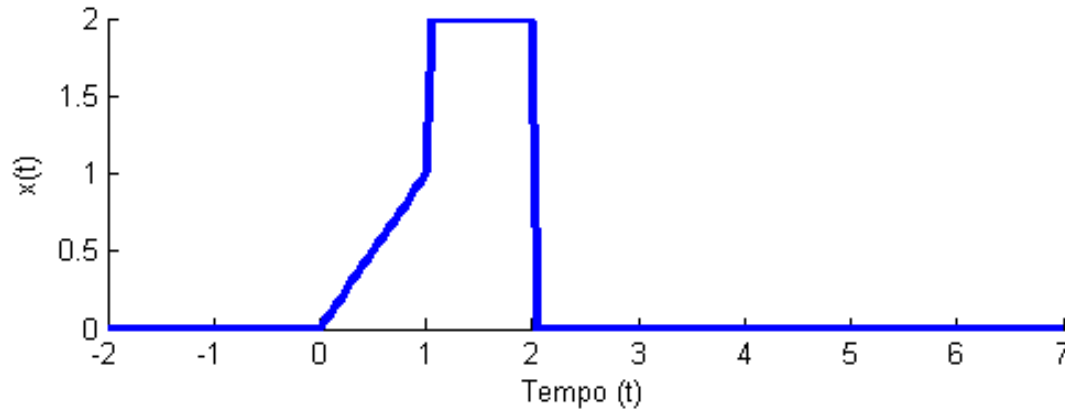
$$x(t) = \begin{cases} 0, & t < 0 \\ t, & 0 \leq t < 1 \\ 2, & 1 \leq t < 2 \\ 0, & 2 \leq t \end{cases}$$

$$h(t) = \begin{cases} 0, & t < -1 \\ 1, & -1 \leq t < 1 \\ e^{-t}, & 1 \leq t < 3 \\ 0, & 3 \leq t \end{cases}$$

Vamos, primeiramente, determinar os intervalos a serem considerados. Em seguida, montaremos as integrais para cada intervalo.

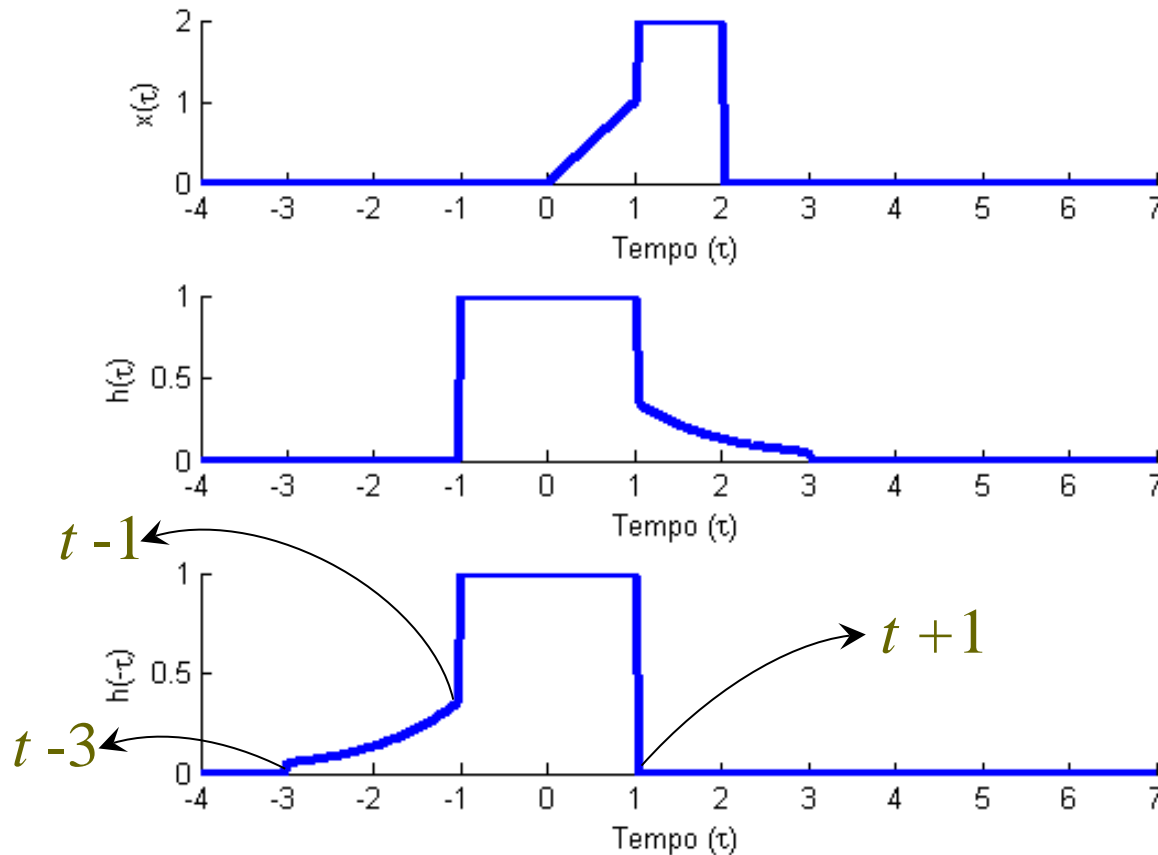
Integral de Convolução

Exemplo: Gráficos da Entrada e da Resposta ao Impulso



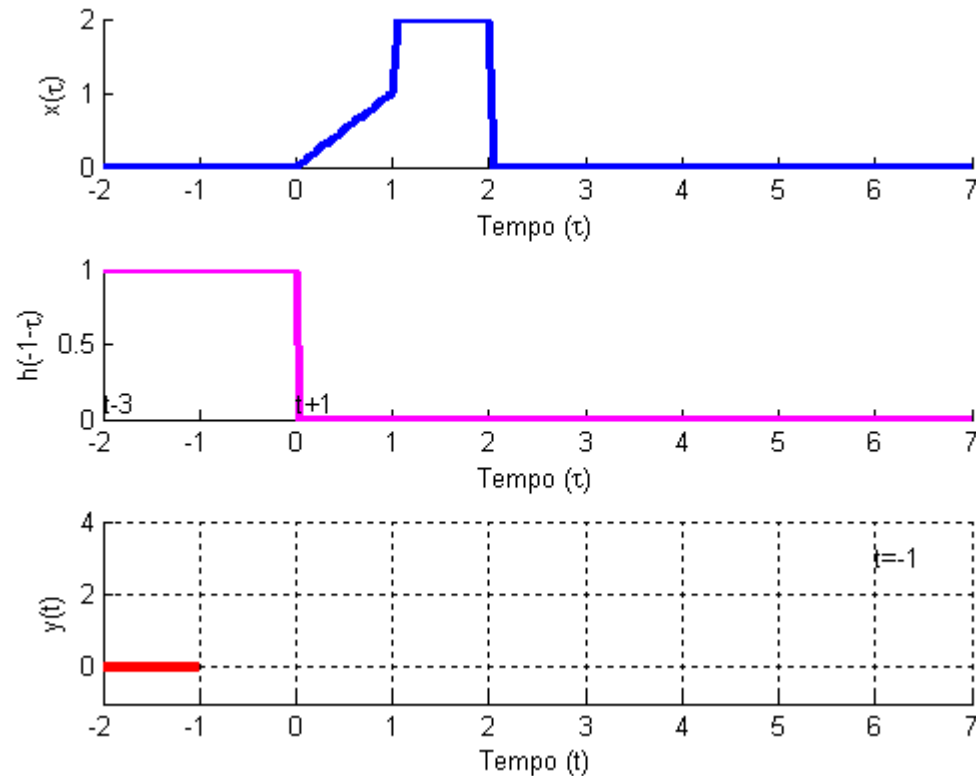
Integral de Convolução

Exemplo: Gráficos da Entrada e da Resposta ao Impulso em τ



Integral de Convolução

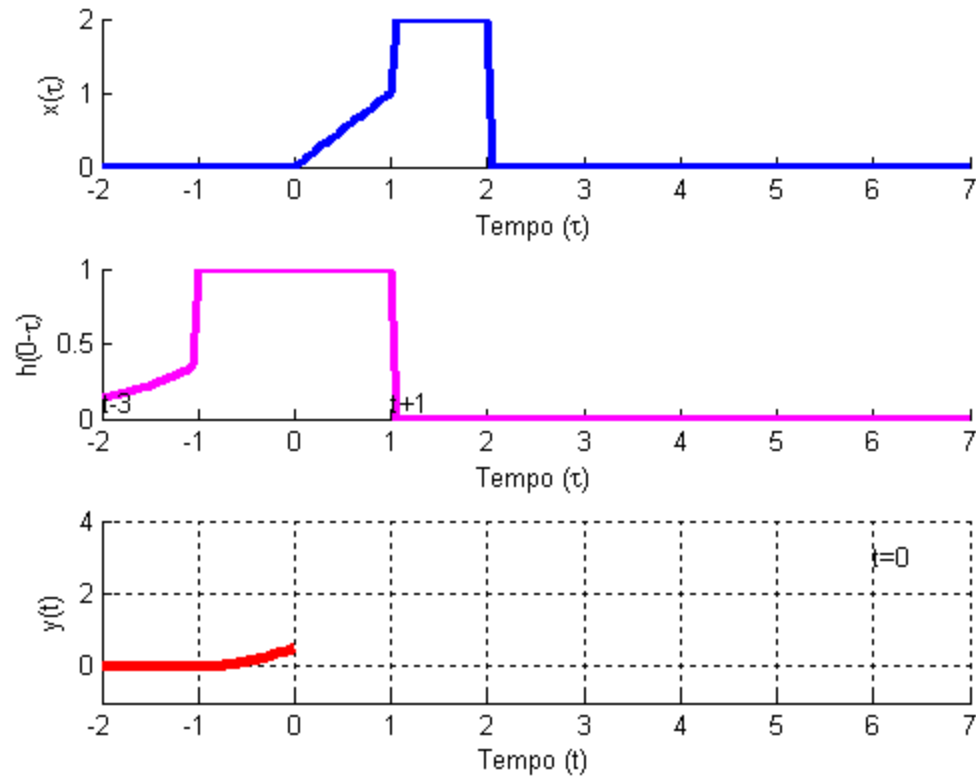
Exemplo: Primeiro Intervalo



$$t < -1 \rightarrow y(t) = 0$$

Integral de Convolução

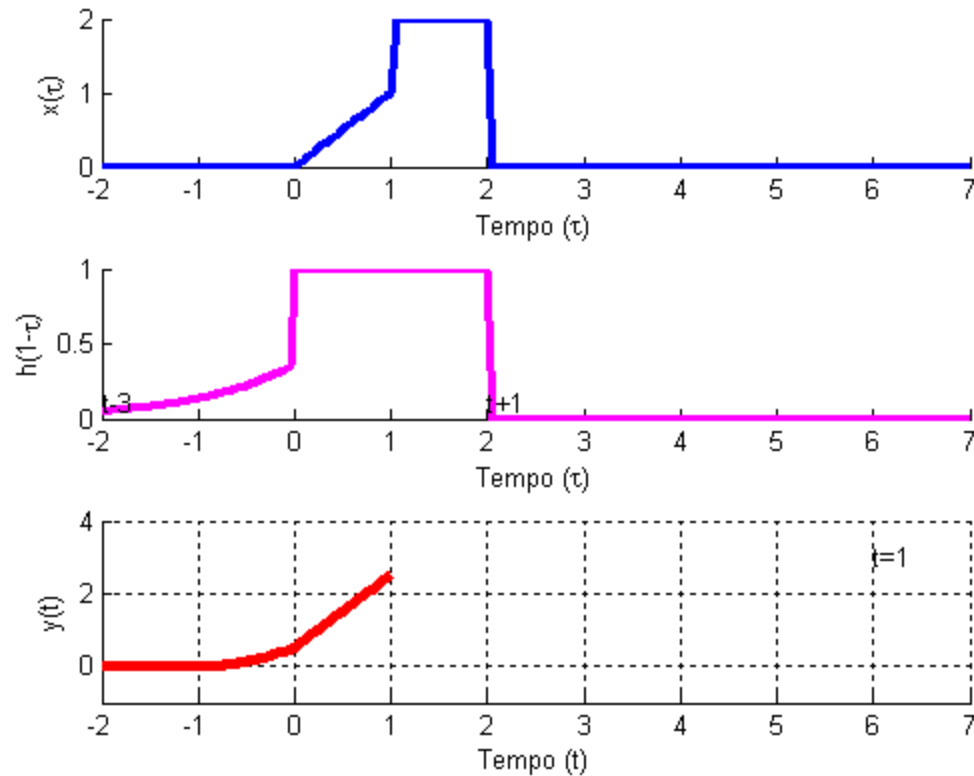
Exemplo: Segundo Intervalo



$$-1 \leq t < 0$$

Integral de Convolução

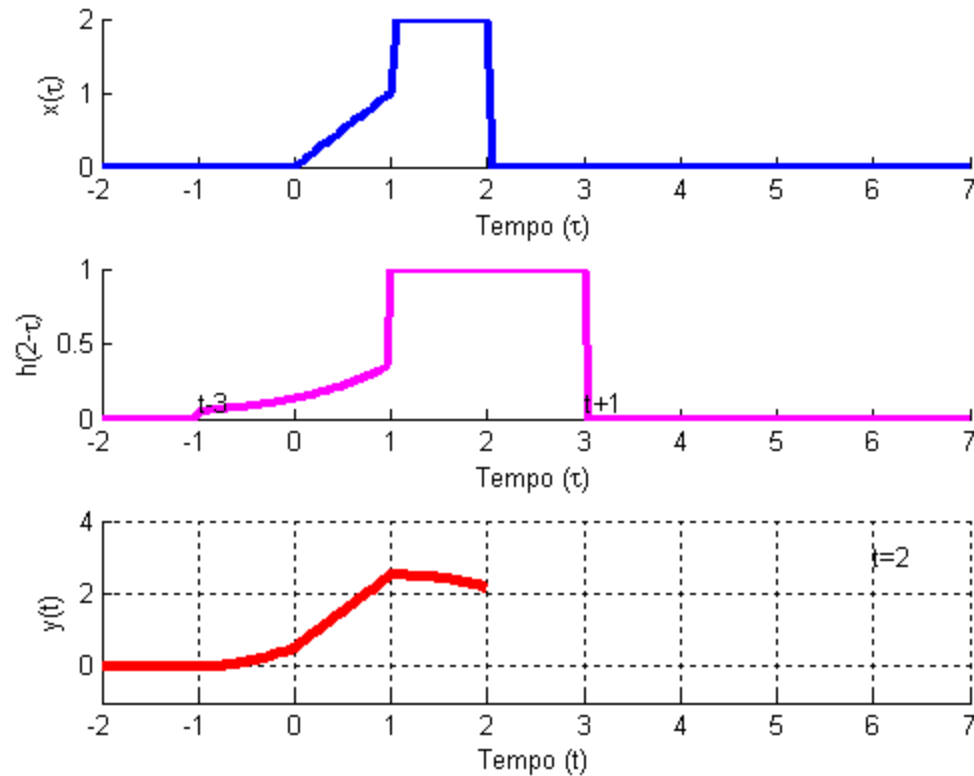
Exemplo: Terceiro Intervalo



$$0 \leq t < 1$$

Integral de Convolução

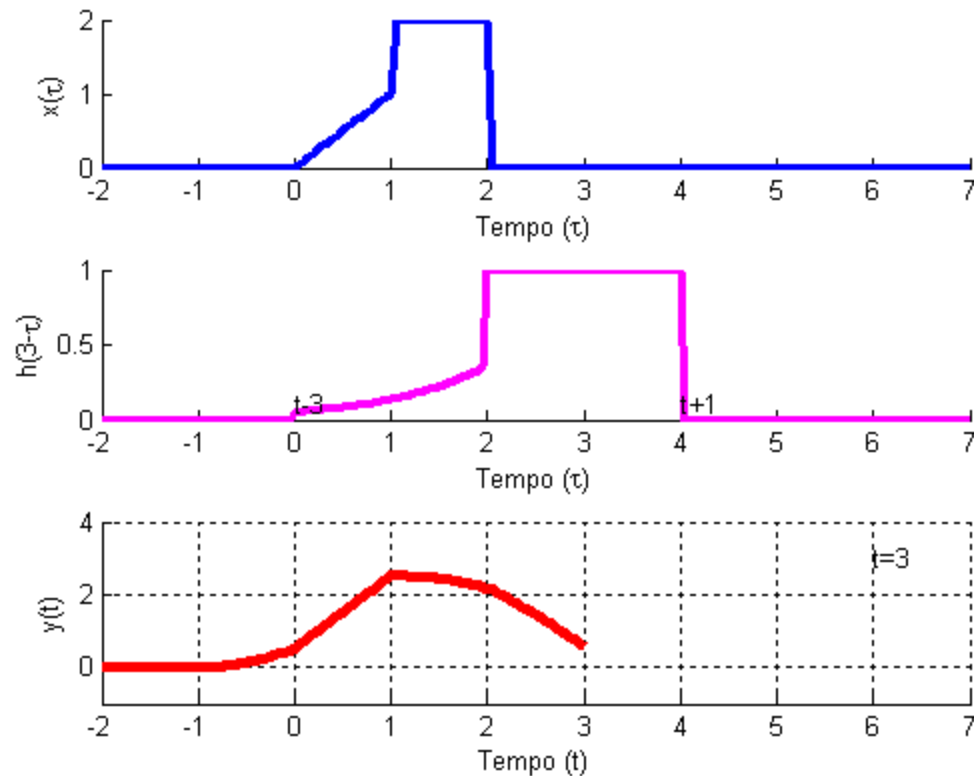
Exemplo: Quarto Intervalo



$$1 \leq t < 2$$

Integral de Convolução

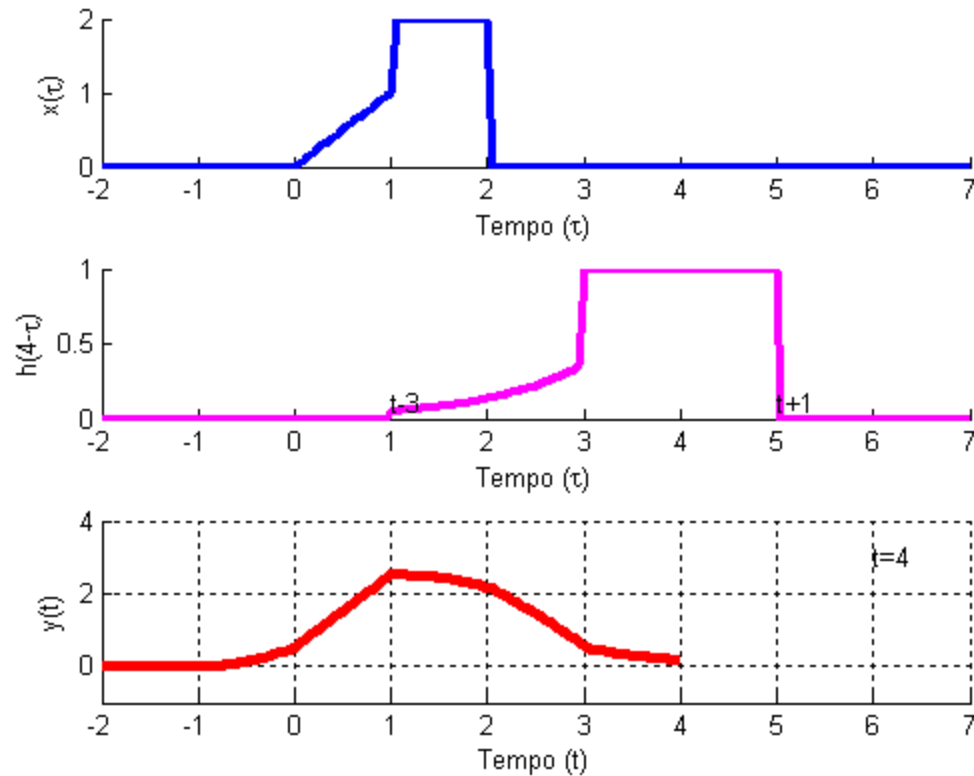
Exemplo: Quinto Intervalo



$$2 \leq t < 3$$

Integral de Convolução

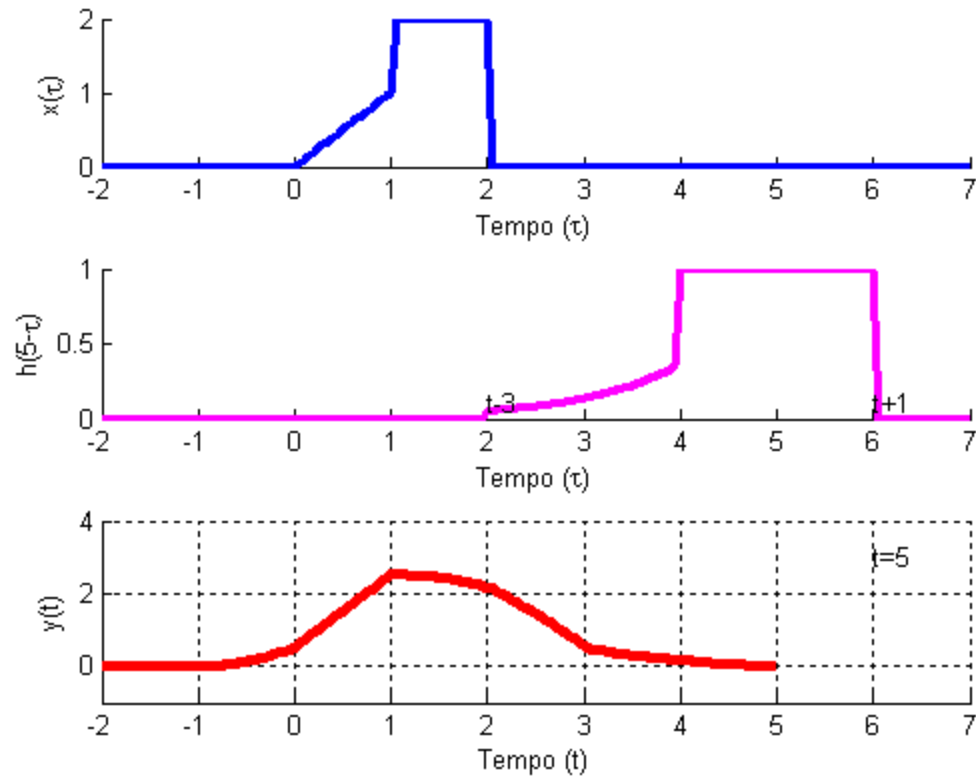
Exemplo: Sexto Intervalo



$$3 \leq t < 4$$

Integral de Convolução

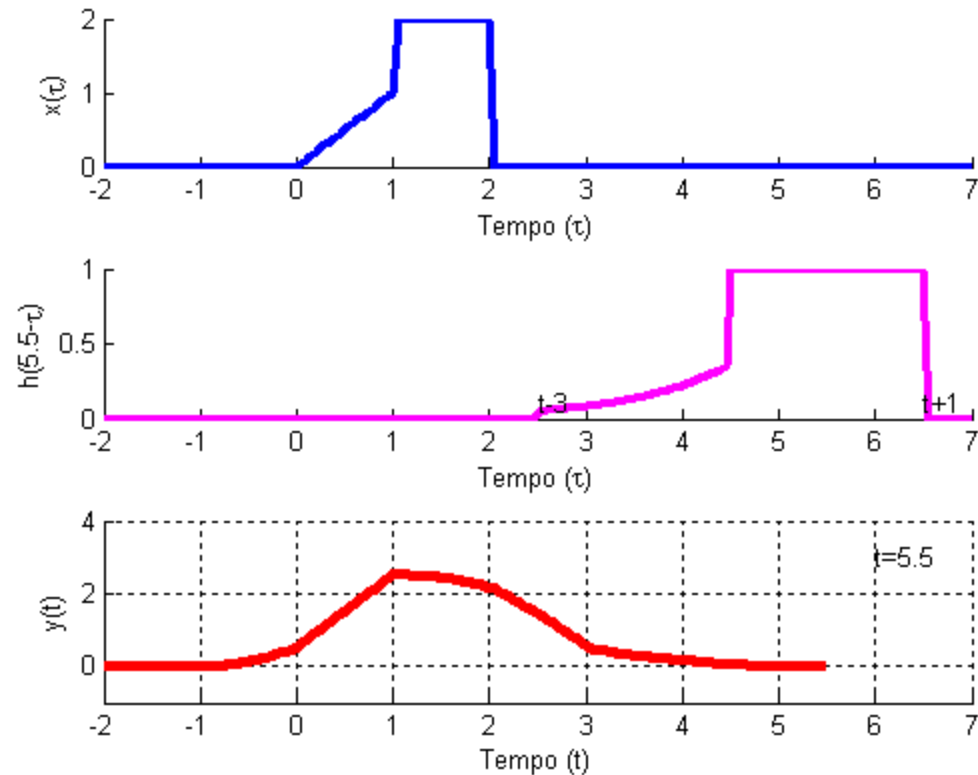
Exemplo: Sétimo Intervalo



$$4 \leq t < 5$$

Integral de Convolução

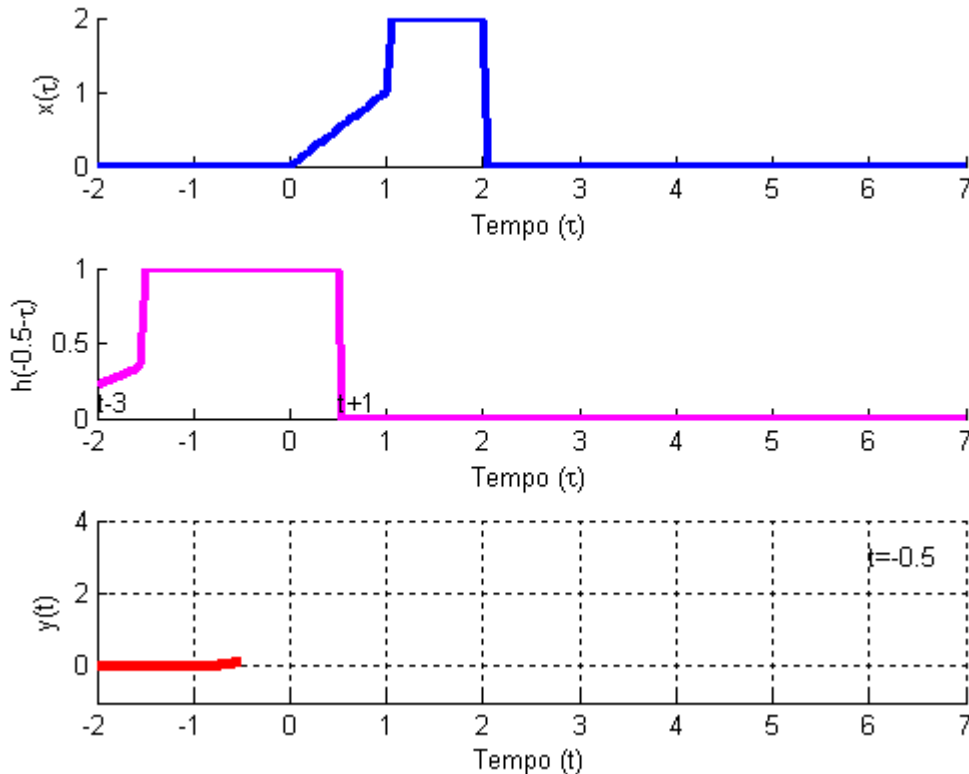
Exemplo: Oitavo Intervalo



$$t > 5 \rightarrow y(t) = 0$$

Integral de Convolução

Exemplo: Segundo Intervalo – Montando a Integral

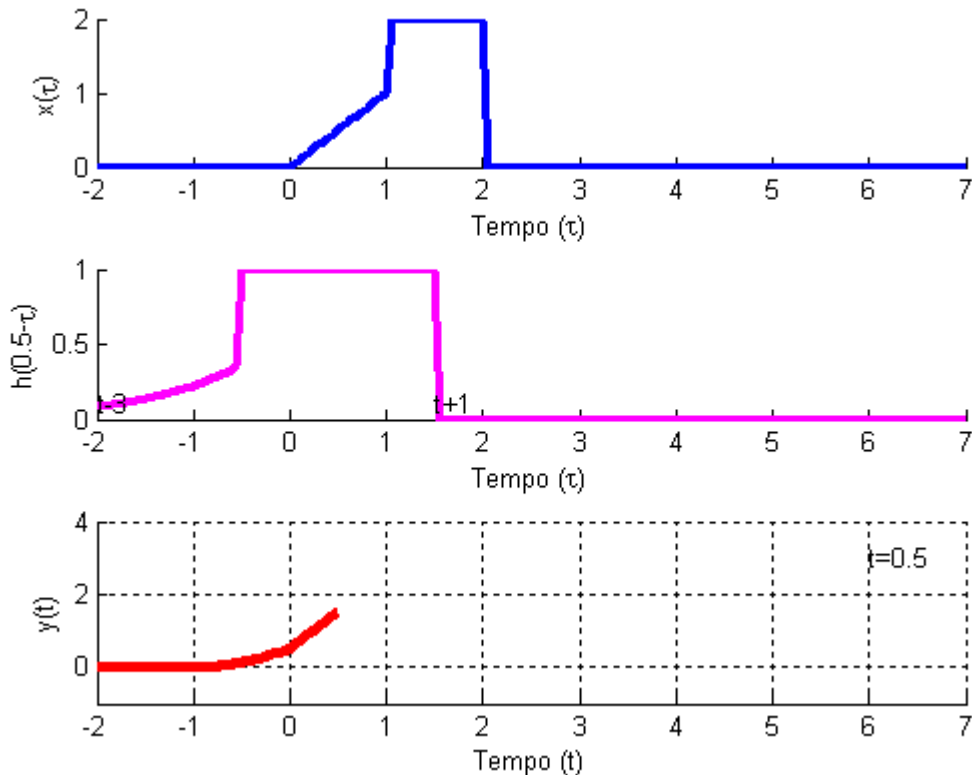


$$y(t) = \int_0^{t+1} \tau d\tau$$

$$-1 \leq t < 0$$

Integral de Convolução

Exemplo: Terceiro Intervalo – Montando a Integral

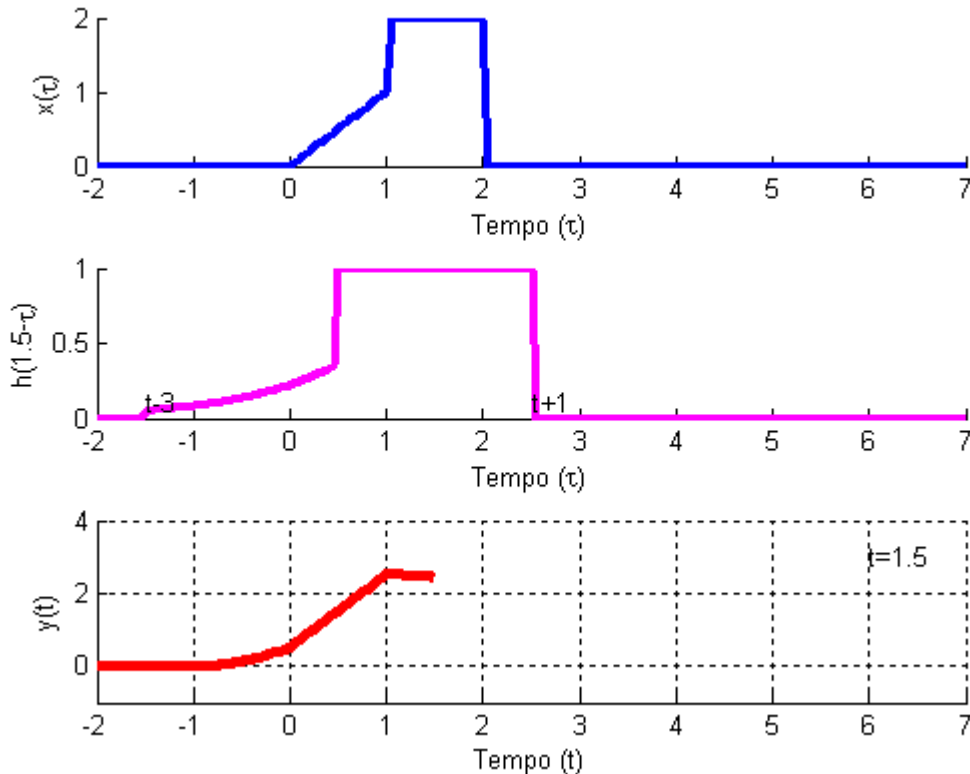


$$y(t) = \int_0^1 1\tau d\tau + \int_1^{t+1} 2d\tau$$

$$0 \leq t < 1$$

Integral de Convolução

Exemplo: Quarto Intervalo – Montando a Integral

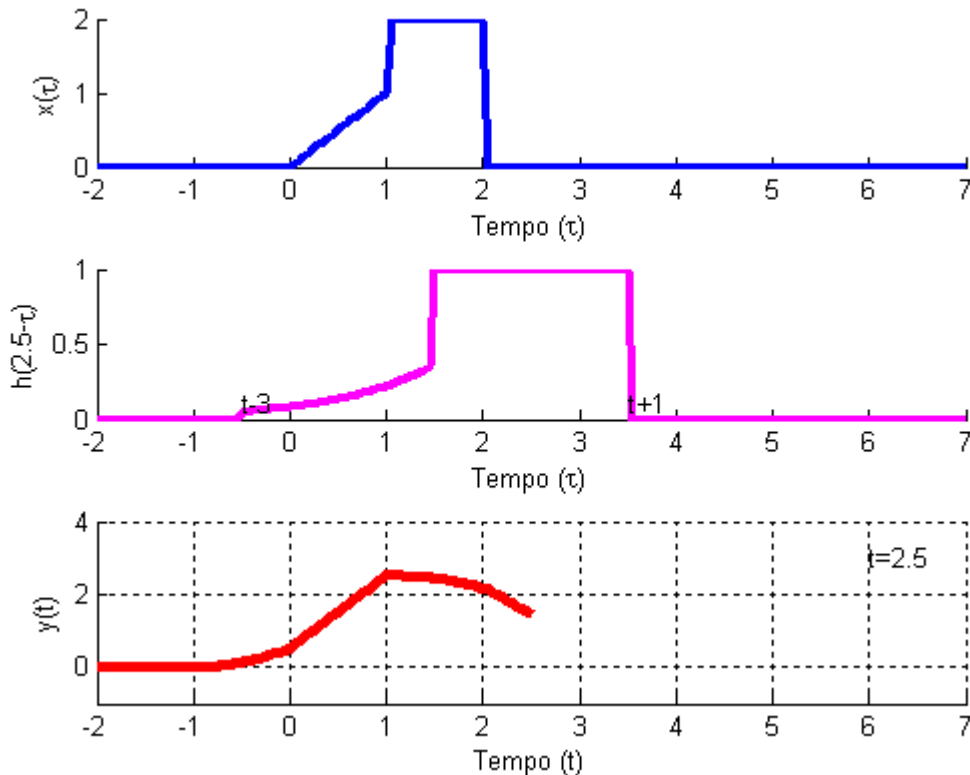


$$1 \leq t < 2$$

$$y(t) = \int_0^{t-1} \tau e^{-(t-\tau)} d\tau + \int_{t-1}^1 \tau d\tau + \int_1^2 2 d\tau$$

Integral de Convolução

Exemplo: Quinto Intervalo – Montando a Integral

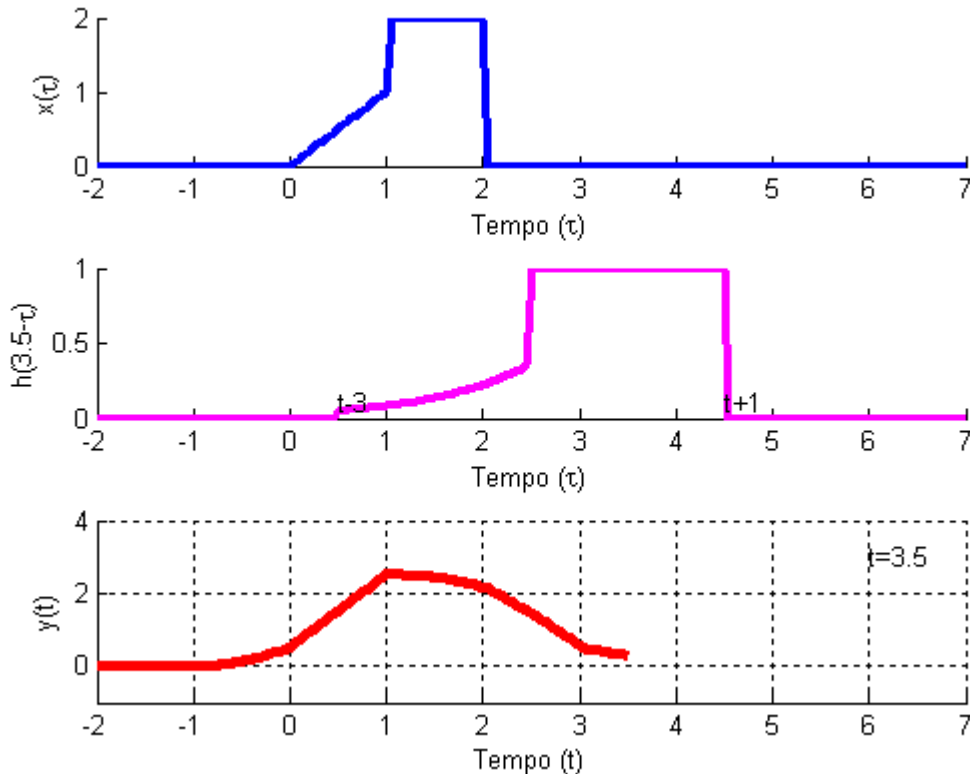


$$y(t) = \int_0^1 \tau e^{-(t-\tau)} d\tau + \int_1^{t-1} 2e^{-(t-\tau)} d\tau + \int_{t-1}^2 2 d\tau$$

$$2 \leq t < 3$$

Integral de Convolução

Exemplo: Sexto Intervalo – Montando a Integral

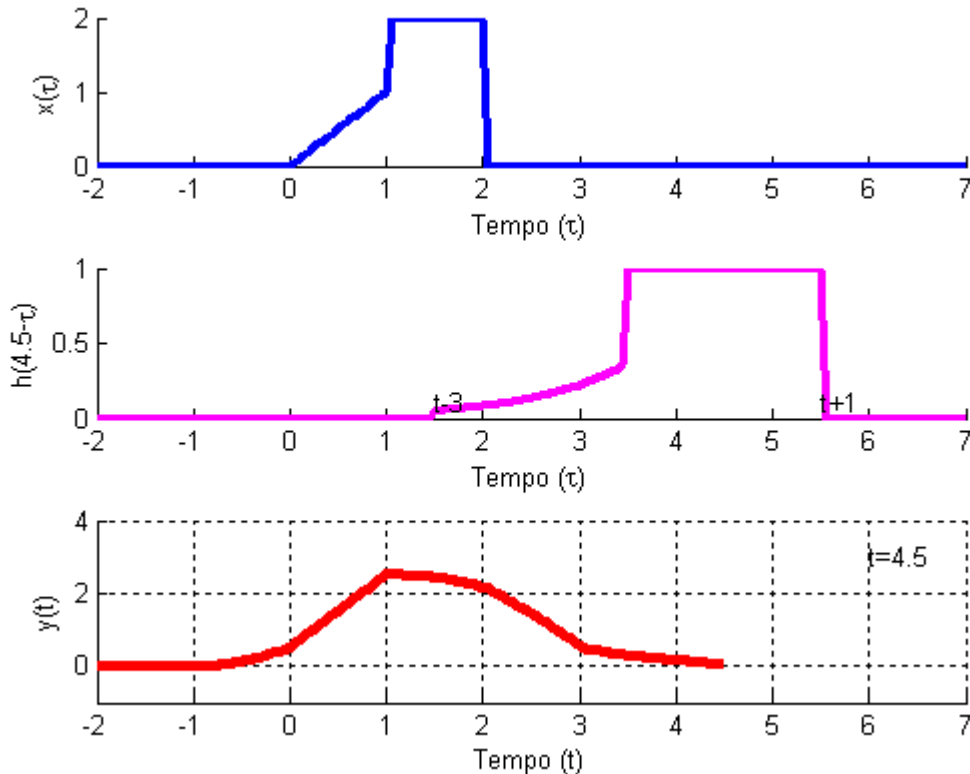


$$y(t) = \int_{t-3}^1 \tau e^{-(t-\tau)} d\tau + \int_1^2 2e^{-(t-\tau)} d\tau$$

$$3 \leq t < 4$$

Integral de Convolução

Exemplo: Sétimo Intervalo – Montando a Integral



$$y(t) = \int_{t-3}^2 2e^{-(t-\tau)} d\tau$$

$$4 \leq t < 5$$

Integral de Convolução

Exemplo: Resumo

Intervalo	$y(t)$
$t < -1$	0
$-1 \leq t < 0$	$\int_0^{t+1} \tau d\tau$
$0 \leq t < 1$	$\int_0^1 1\tau d\tau + \int_1^{t+1} 2d\tau$
$1 \leq t < 2$	$\int_0^{t-1} \tau e^{-(t-\tau)} d\tau + \int_{t-1}^1 \tau d\tau + \int_1^2 2d\tau$

Integral de Convolução

Exemplo: Resumo

Intervalo	$y(t)$
$2 \leq t < 3$	$\int_0^1 \tau e^{-(t-\tau)} d\tau + \int_1^{t-1} 2e^{-(t-\tau)} d\tau + \int_{t-1}^2 2d\tau$
$3 \leq t < 4$	$\int_{t-3}^1 \tau e^{-(t-\tau)} d\tau + \int_1^2 2e^{-(t-\tau)} d\tau$
$4 \leq t \leq 5$	$\int_{t-3}^2 2e^{-(t-\tau)} d\tau$
$t > 5$	0

Integral de Convolução

Exemplo de Novo – MS2P4.m (Sinais e Sistemas Lineares – B. P. Lathi, Segunda Edição)

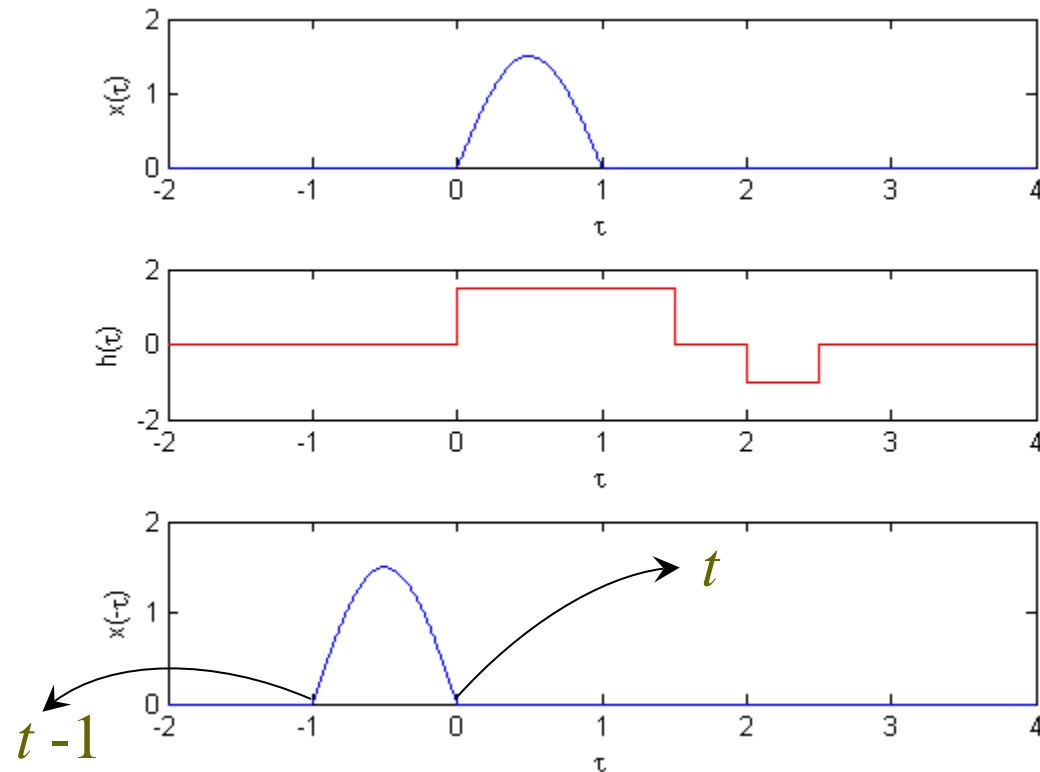
$$x(t) = \begin{cases} 1.5 \operatorname{sen}(\pi t) & 0 \leq t < 1 \\ 0 & \textit{caso contrário} \end{cases}$$

$$h(t) = \begin{cases} 1.5 & 0 \leq t < 1.5 \\ -1 & 2 \leq t < 2.5 \end{cases}$$

Na seção Material Extra, da página da disciplina, estão disponibilizados os programas para Matlab do livro Sinais e Sistemas Lineares, B. P. Lathi, Segunda Edição.

Integral de Convolução

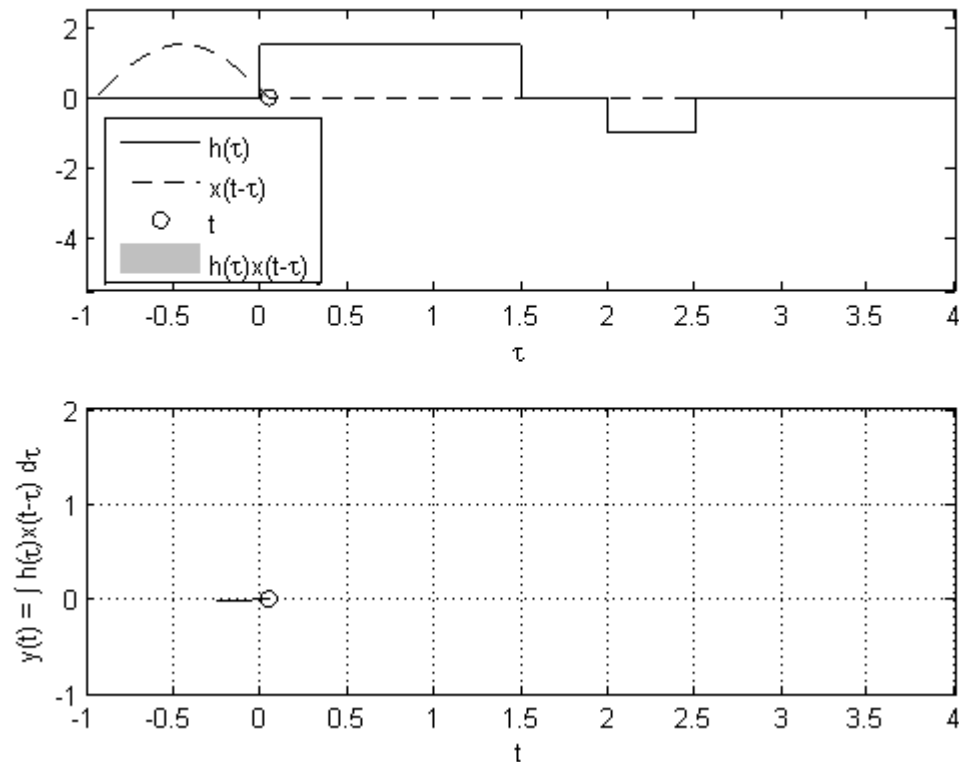
Exemplo: Gráficos da Entrada e da Resposta ao Impulso em



Vamos rebater e deslocar a entrada...

Integral de Convolução

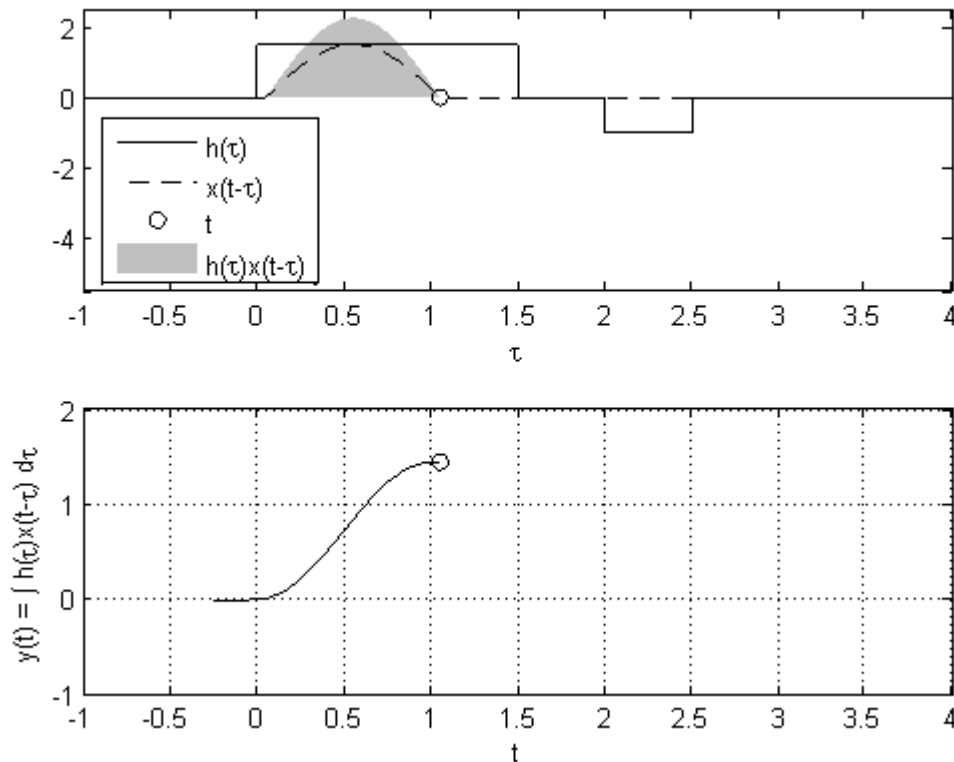
Exemplo: Primeiro Intervalo



$$t < 0 \rightarrow y(t) = 0$$

Integral de Convolução

Exemplo: Segundo Intervalo

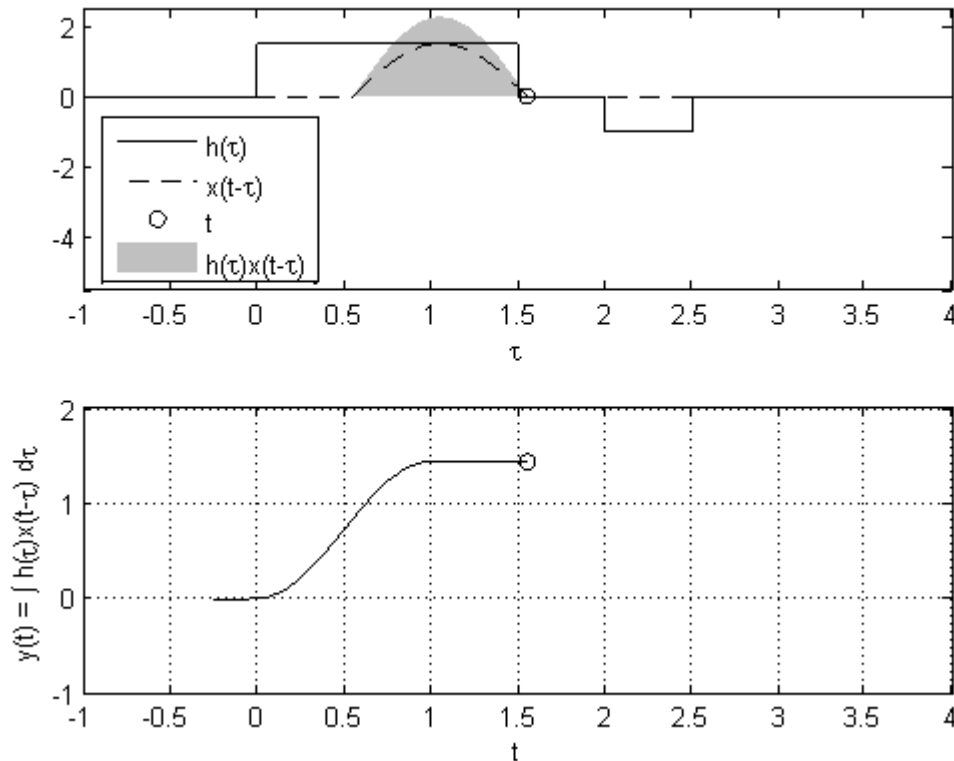


$$y(t) = \int_0^t 1.5 \text{sen}(\pi(t - \tau))(1.5) d\tau$$

$$0 \leq t < 1$$

Integral de Convolução

Exemplo: Terceiro Intervalo

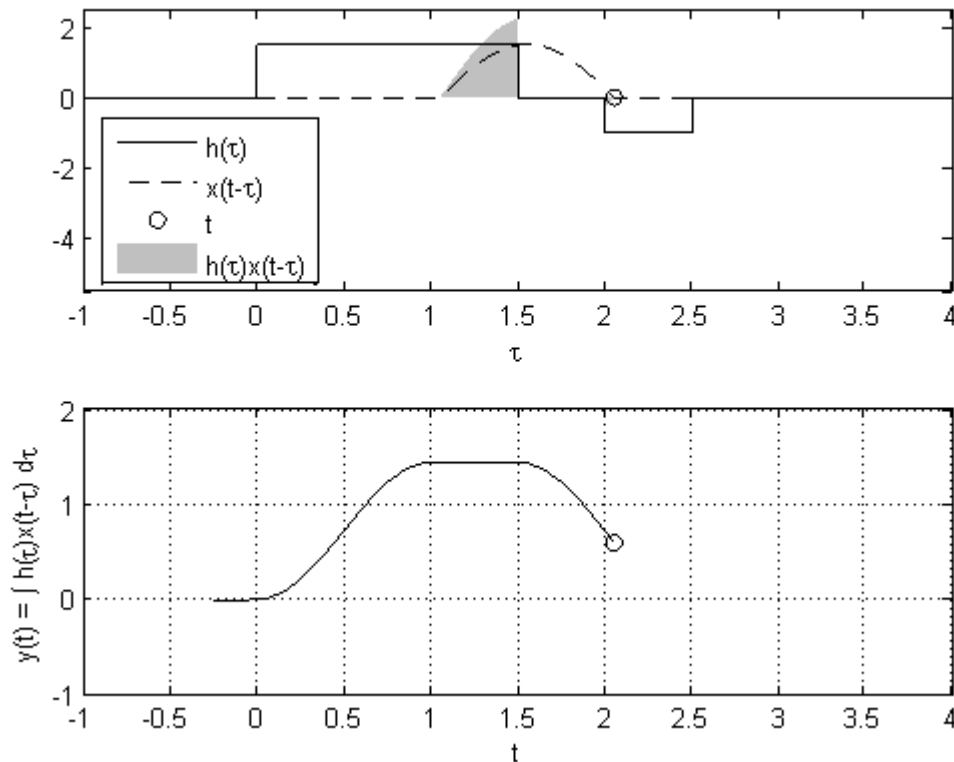


$$y(t) = \int_{t-1}^t 1.5 \text{sen}(\pi(t-\tau))(1.5) d\tau$$

$$1 \leq t < 1.5$$

Integral de Convolução

Exemplo: Quarto Intervalo

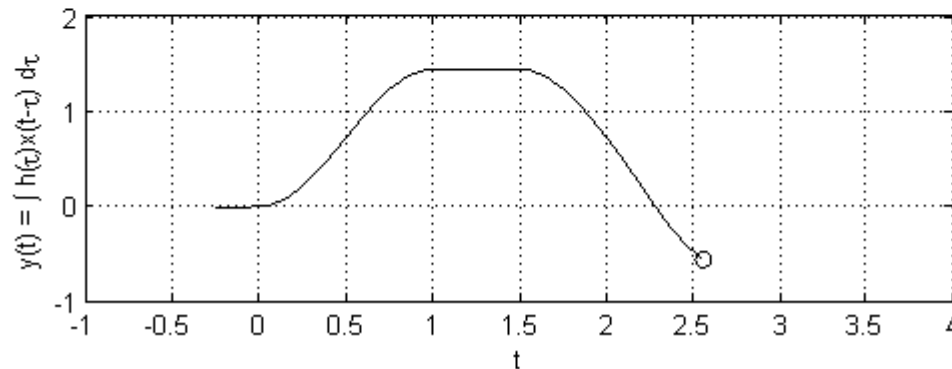
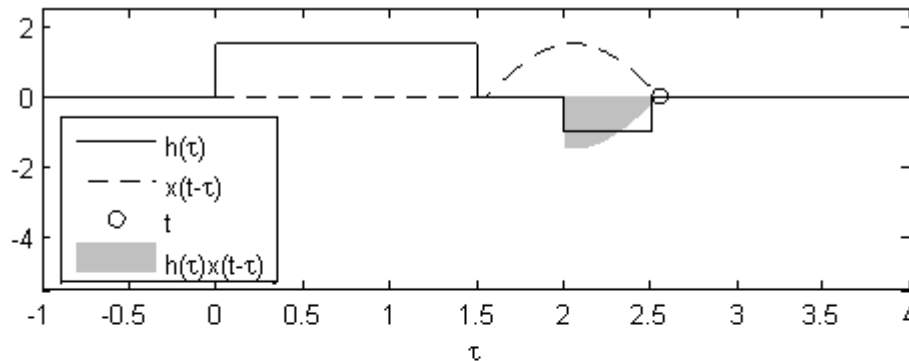


$$y(t) = \int_{t-1}^{1.5} 1.5 \text{sen}(\pi(t-\tau))(1.5) d\tau$$

$$1.5 \leq t < 2$$

Integral de Convolução

Exemplo: Quinto Intervalo

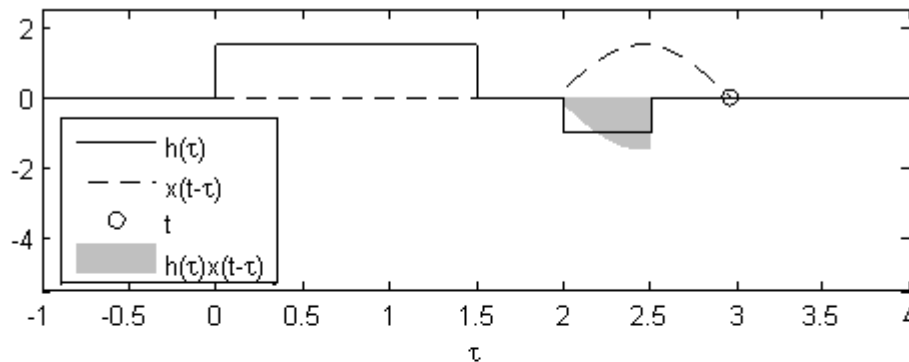


$$2 \leq t < 2.5$$

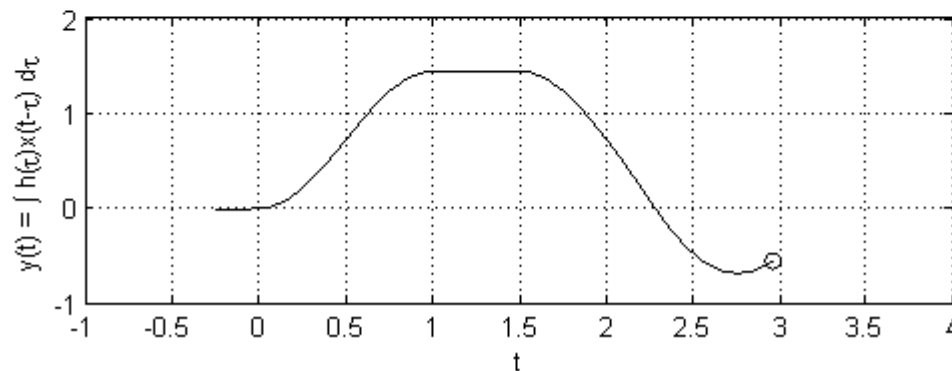
$$y(t) = \int_{t-1}^{1.5} 1.5 \text{sen}(\pi(t-\tau)) 1.5 d\tau + \int_2^t 1.5 \text{sen}(\pi(t-\tau)) (-1) d\tau$$

Integral de Convolução

Exemplo: Sexto Intervalo



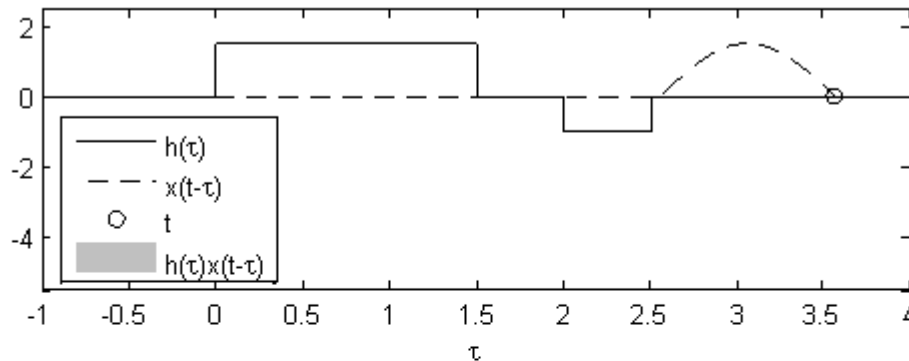
$$y(t) = \int_2^{2.5} 1.5 \text{sen}(\pi(t - \tau))(-1) d\tau$$



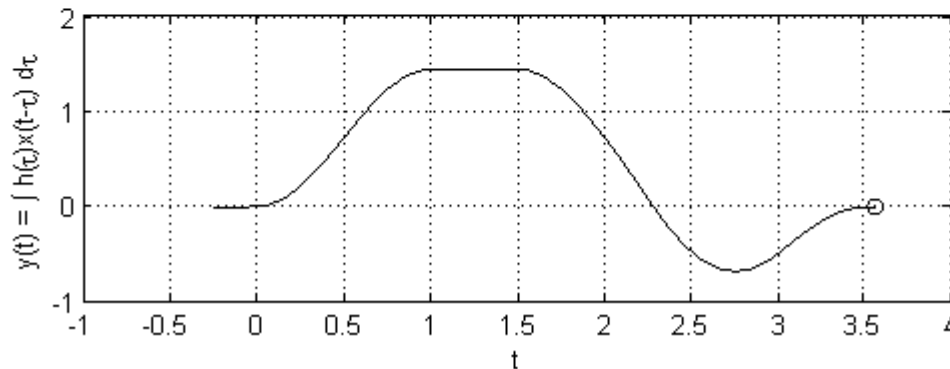
$$2.5 \leq t < 3$$

Integral de Convolução

Exemplo: Sexto Intervalo



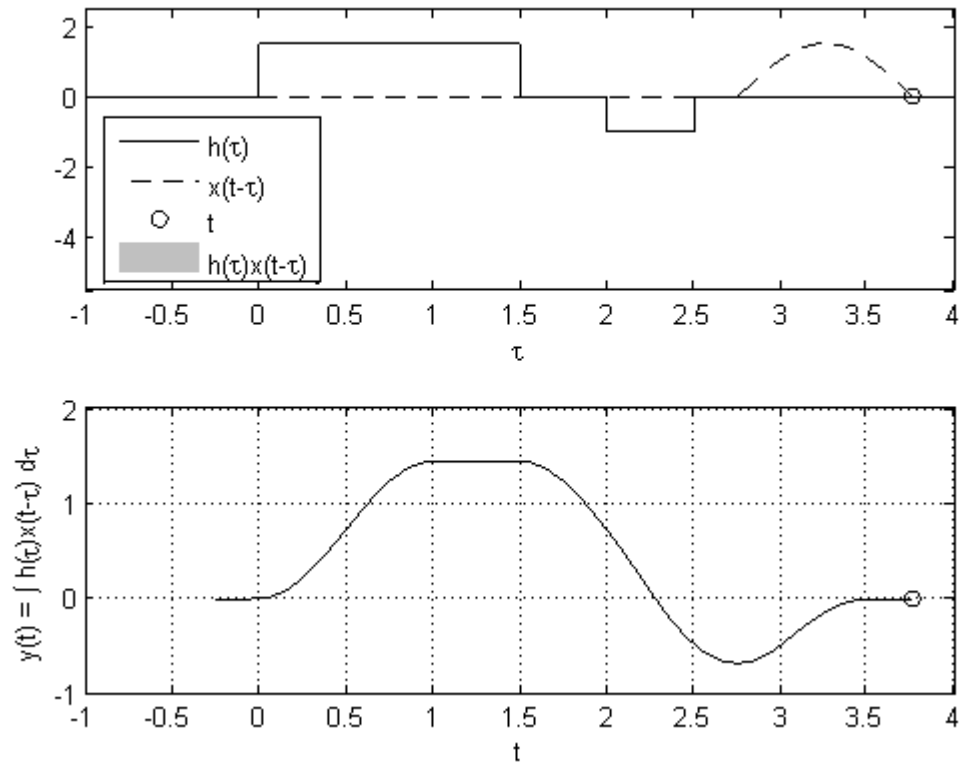
$$y(t) = \int_{t-1}^{2.5} 1.5 \text{sen}(\pi(t-\tau))(-1) d\tau$$



$$3 \leq t < 3.5$$

Integral de Convolução

Exemplo: Sétimo Intervalo



$$t > 3.5 \rightarrow y(t) = 0$$

Integral de Convolução

Exemplo: Resumo

Intervalo	$y(t)$
$t < 0$	0
$0 \leq t < 1$	$\int_0^t 1.5 \text{sen}(\pi(t - \tau))(1.5) d\tau$
$1 \leq t < 1.5$	$\int_{t-1}^t 1.5 \text{sen}(\pi(t - \tau))(1.5) d\tau$
$1.5 \leq t < 2$	$\int_{t-1}^{1.5} 1.5 \text{sen}(\pi(t - \tau))(1.5) d\tau$

Integral de Convolução

Exemplo: Resumo

Intervalo	$y(t)$
$2 \leq t < 2.5$	$\int_{t-1}^{1.5} 1.5 \text{sen}(\pi(t-\tau))(1.5) d\tau + \int_2^t 1.5 \text{sen}(\pi(t-\tau))(-1) d\tau$
$2.5 \leq t < 3$	$\int_2^{2.5} 1.5 \text{sen}(\pi(t-\tau))(-1) d\tau$
$3 \leq t \leq 3.5$	$\int_{t-1}^{2.5} 1.5 \text{sen}(\pi(t-\tau))(-1) d\tau$
$t > 3.5$	0

Boa Notícia!

VOCÊS JÁ PODEM FAZER A QUARTA
LISTA DE EXERCÍCIOS SUGERIDOS...